

Optimal Use of Financial Information

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This paper investigates the conditions under which a possessor of valuable information on financial security may prefer to sell it directly or give it away free of charge *i.e.*, donating it to other market participants instead of trading on it. A market participant will never find it optimal to sell or donate the information that s/he has monopolistic ownership of. Otherwise, sale or donation of information has an important commitment effect in that it credibly commits a risk neutral possessor of information to a strategy which promotes more intense competition among informed traders in the market and makes the trading strategies of other informed traders less aggressive. It is this strategic externality that makes the selling or donation of information an optimal strategy. The model in this paper also shows that if the security price does not fully reflect the private information of all the traders, diluting the seller's information before selling it is not optimal even if the seller trades on her own account while selling her information.

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JEL Classification: D82, G14, L13

I. Introduction

Economic agents participate in the trading of financial securities for various reasons, most notably for the purpose of risk sharing or earning profits, and the value of information on financial securities that these agents might have private access to cannot be emphasized too much. This paper investigates three possible use of private information on financial securities for the benefit of its possessor: trading on it, selling directly to other market participants who do

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not own it, and giving it away to them. The third of using financial information is called donation of information since unlike direct sales of financial information, private information is given away free of charge to any market participants who are willing to take it.

I refer to direct sale of information as any case in which the end user of the information gets to observe the information before he decides to act on it. The sale of information by a mechanism like the subscription to an investment newsletter of limited circulation is a typical example of this way of selling information. Thus, delegated portfolio management through a mutual fund manager for a fee is not covered by the definition of a direct sale mechanism.¹ The typical question that confronts an information seller in a financial market is why she has to resort to the sale or donation of information if she could directly trade on it herself and, presumably, make greater profits thereby. This paper investigates the conditions under which a possessor of valuable information may prefer to sell or donate her information instead of trading on her own account. In addition, this paper explores the nature of the optimal sales strategy under different structures.

First, it is established that for a risk neutral possessor of information who has monopolistic access to information about a financial security, it is never optimal to sell or donate it instead of trading on it. This conclusion is based on the assumption that trading in securities can be achieved in an anonymous fashion. Once the strong assumption of monopolistic access to information is removed, it may no longer be optimal for an information possessor to abjure the direct sale of her information. In fact, the sale of information to clients who will then optimally use the information to decide their trading strategies has important effects on the nature of trading in the financial markets. Such a sale of information has important commitment effects in that it credibly commits the information possessor to a strategy that would not be credible if she were to avoid such sales. Intuitively, the sale of information to a number of clients provides for more intense competition in the financial market. While this does reduce the total profits available to informed traders as a group, it also has the effect of making the trading strategies of other information-

¹Please refer to Bhattacharya and Pfleiderer (1985) for the analysis of delegated portfolio management.

based traders less aggressive. As a result, although the reduction in overall profits from trading may be substantial, the individual seller of information does not bear the full cost of the reduction in overall profits. This strategic externality may make the selling of information an optimal strategy for the possessor of information.

Extreme case of direct sales of information is simply giving away free of charge *i.e.*, donating private information on financial security. Any market participants purchasing information from the information seller expect to earn trading profit from it, and the information seller is extracting all or part of the information buyers' trading profit by charging a positive price. It is shown that if the information seller is able to extract only small portion of the information buyers' trading profit, and the profit from the sales of information constitutes a relatively minor part of the seller's total from her information, then strategic effect previously discussed will be maximized by the donation of information.

The issue of the selling information in the context of financial markets has been analyzed by Admati and Pfleiderer (1986, 1988a) in two papers. In Admati and Pfleiderer (1986), they show that in a competitive rational expectations setup, the optimal way to sell information is to make it coarser by means of adding '*personalized noise*' to the information. This addition of noise prevents full revelation of information by the market price in the rational expectations equilibrium, and thus preserves the value of private information. In Admati and Pfleiderer (1988a), they show that it may be optimal for a monopolistic risk averse information possessor to sell her information in order to achieve better risk sharing.

The analysis in this paper yields results that are different from those of Admati and Pfleiderer, due to the use of a strategic model of financial market trading. In such a model, given the specified sequence of moves on the part of various players, the final price never reveals the information in full as in a competitive rational expectations framework, and it may indeed be optimal to sell information even without appealing to risk sharing considerations. In particular, this paper also shows that it is optimal for the seller never to dilute her information by the addition of noise, whether '*personalized*' or not, even if she trades on her own account as well. Thus, the results in this paper mitigate the objection that the optimal strategy established may be illegal due to discrimination amongst customers.

Fishman and Hagerty (1995) and Sabino (1993) also investigate the incentive for the sales of information. There are two major differences between Fishman and Hagerty (1995) and this paper. Firstly, in Fishman and Hagerty (1995), the sequence of game is as follows: (i) the market maker chooses the price schedule; (ii) given the price schedule information seller decides the optimal information selling strategy; and (iii) trading of securities commences. In this sequence of game, no matter what sales strategy is taken by the information seller, the price schedule chosen by the market maker does not change and consequently market liquidity is not affected by the sales of information. However, in this paper and Sabino (1993), the first two stages are reversed, and considering the effect of information sales on the market liquidity, the information seller optimally decides the strategy of information sales. As will be shown in the following section, the condition for the information sales are not affected by the sequence of game. Secondly, and more importantly, the model in this paper has fewer restriction than Fishman and Hagerty (1995) and Sabino (1993). This paper demonstrates that diluting the seller's information before selling it is not optimal even if the seller trades on her own account while selling her information. Therefore, the results derived in this paper are a lot stronger than those from Fishman and Hagerty (1995) and Sabino (1993).

As mentioned above, this paper does not deal with the moral hazard aspect of the sales of financial information. That issue is the focus of a paper by Allen (1990), in which he shows that in order to convince the buyer of the veracity of the information, the seller may have to make her own wealth contingent on the price outcome of the security about which she claims to have private information. The model in this paper, on the other hand, is based on the assumption that the buyers of information can costlessly verify whether the seller has engaged in adequate information gathering or not, although the precise outcome of the investigative process is not directly observable to the buyer. The analysis in this paper is related to that of Kamien and Tauman (1986), in which the single patent holder of a cost reducing innovation in a product market finds it optimal to license unless he is a monopolist in the product market. Kane and Marks (1990) and Brennan and Chordia (1993) compare direct sales of information to other methods of indirect sales of information. Kane and Marks (1990) shows that in

the presence of borrowing constraints, investors prefer direct sales of information. In Brennan and Chordia (1993), different ways to charge customers of information sales are compared.

This paper is also related to the work of Bushman and Indjejikian (1995), in which the authors model a manager and market analysts who have *costly* access to information on the same random variable. They show that managers will disclose a noisy signal of their information to discourage the analysts from getting any information. Disclosure in their model reduces the potential profit for analysts and, thus, acts like an entry deterrent.²

The rest of this paper is organized in four sections. Section II presents the basic model of the financial market which will be used throughout the paper. The model is an adaptation of the model in Kyle (1985), and the condition for the sales of information is derived. The basic model introduced in Section II will be generalized in Section III. The case of donation of information is analyzed in Section IV. Section V discusses directions for future research and conclusions. All proofs are presented in Appendix.

II. The Model

A single risky security is traded in a financial market. The *ex post* payoff of this security, denoted \tilde{v} , is normally distributed with mean \bar{v} . The innovation from the *ex ante* expectation of \tilde{v} is composed of two components: the value of existing assets (*i.e.*, asset component), denoted $\tilde{\theta}$, and the present value of growth opportunities (*i.e.*, growth component), denoted $\tilde{\varepsilon}$ respectively. Specifically,

$$\tilde{v} = \bar{v} + \tilde{\theta} + \tilde{\varepsilon}. \quad (1)$$

$\tilde{\theta}$ and $\tilde{\varepsilon}$ are mutually independent and normally distributed with mean 0³ and variances σ_{θ}^2 and σ_{ε}^2 respectively.

There are five different types of agents participating in the

²The intuition of reducing the profits on entry is not very different from the well known entry deterrence models in the industrial organization literature.

³We assume a zero mean for simplification purposes. All our results go through even if these random variables, instead, have a positive mean.

financial market. A monopolistic *information seller*⁴ has costless access to a private observation of both asset and growth components without any noise.⁵ The information seller can trade on her own account, sell her information, or do both.

We assume that the nature of the growth component is such that market participants other than the information seller are unable to obtain superior information about its innovation. However, there are N *market analysts* who are able to ascertain the innovation in the asset component of the this risky security without any noise. Any traders who trade on the information obtained by studying the market by themselves are termed analysts, and they are not allowed to buy information from the information seller. Arbitrageurs and fund managers working for brokerage firms and investment banks, and even insiders, are included in this group.

In addition to the analysts and the information seller, there also exist liquidity traders who trade for reasons exogenous to the model, and a competitive market maker who set prices that give them zero expected profits conditional on their information. The demand by the liquidity traders, which gets aggregated with the demand by other traders, is \tilde{u} , and it is independent of $\tilde{\theta}$ and $\tilde{\varepsilon}$, and normally distributed with mean 0 and variances σ_u^2 .

Sufficiently many *outside investors* have neither information about \tilde{v} nor any liquidity demand for the security, and they are potential clients of the information seller.

We model the mechanism of information sales as private provision of garbled signal of the seller's information. Specifically, we assume that the seller has access to the random variables $\tilde{\mu}_i$'s and $\tilde{\omega}_j$'s, which can be used for garbling the signal before selling it to the outside investors.⁶ Thus, if the information seller decides to

⁴This paper does not model how the information seller has obtained monopolistic right to sell her information. She might possess exclusive ownership of the technology that can credibly convey her information to would-be buyers, or government simply bars any market participants other than this information seller from selling their information to potential buyers.

⁵The results do not change as far as the information acquisition cost to be incurred by the information seller is sufficiently low that it is always optimal for him to acquire costly information.

⁶This method of garbling information is similar to the one generally modelled in the literature. See, for example, Admati and Pfleiderer (1986)

sell the information on $\tilde{\theta}$ to K outside investors with degree of noise (variance) σ_{μ}^2 , then each outside investor who purchases information on $\tilde{\theta}$ from the seller privately observes following signal,

$$\tilde{s}(\theta)_i = \tilde{\theta} + \tilde{\mu}_i, \quad i = 1, 2, \dots, K. \quad (2)$$

On the other hand, if information on $\tilde{\varepsilon}$ is sold to M outside investors with degree of noise, σ_{ω}^2 , then each buyer of information on $\tilde{\varepsilon}$ has private access to following signal,

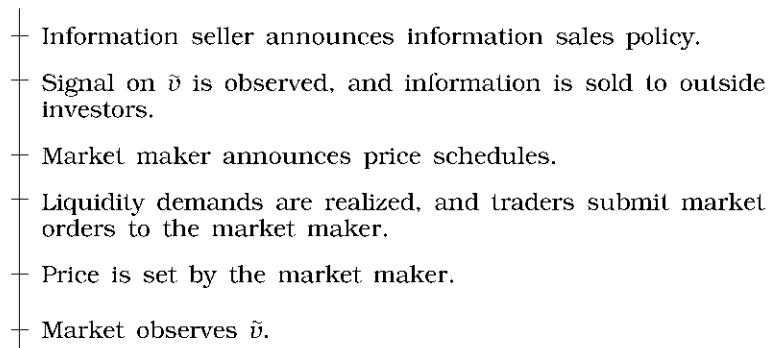
$$\tilde{s}(\varepsilon)_j = \tilde{\varepsilon} + \tilde{\omega}_j, \quad j = 1, 2, \dots, M. \quad (3)$$

Although information seller has perfect information on $\tilde{\theta}$ and $\tilde{\varepsilon}$, it is assumed that the random variables $\tilde{\mu}_i$'s and $\tilde{\omega}_j$'s that are used for garbling $\tilde{\theta}$ and $\tilde{\varepsilon}$ before selling it to the outside investors cannot be observed by the information seller.⁷ Information seller precommits following aspects of information sales policy before any signal on \tilde{v} is observed: the number of buyers of each component of \tilde{v} , and the levels of noise to be added to $\tilde{\theta}$ and $\tilde{\varepsilon}$.

Sequence of trading is given in Figure (1). Information seller announces information sales policy before any trader observes signal on the payoff of the security. After information seller and

and Bushman and Indjejikian (1995).

⁷This way of garbling signal before selling to the buyers is called adding 'personalized noise' by Admati and Pfleiderer (1986) explaining that "Signals may be personalized in other, less direct ways. For example, the seller may provide information that is vague and open to interpretation, so that the buyers themselves make personal, independent, errors of interpretation." If noise is added in this way, each buyer interpret the information provided by the seller differently and the seller cannot possibly know how each buyer interprets the information. As information is sold in this way, each information buyer observes different signal. Actual process of garbling signal can be contemplated in the following way. Instead of letting information buyers know exact value of $\tilde{\theta}$ and $\tilde{\varepsilon}$, information seller just provides basic information that are needed for the valuation of $\tilde{\theta}$ and $\tilde{\varepsilon}$. It is up to each information buyer to interpret the information and conduct valuation in his or her own way. Then, depending on the way in which information is interpreted and what method of valuation is adopted, information buyer will reach different valuation of $\tilde{\theta}$ and $\tilde{\varepsilon}$, that cannot be directly observed by the information seller. As more detailed information for the valuation of $\tilde{\theta}$ and $\tilde{\varepsilon}$ is provided by the information seller, the level of noise is reduced accordingly.

**FIGURE 1**

TIMELINE

analysts observe signals on \tilde{v} , information is sold to outside investors as precommitted by the seller, and subsequently outside investors who just purchase information from the seller privately observe the signals on $\tilde{\theta}$ or $\tilde{\varepsilon}$. Then, market maker announces price schedule, and trading commences. Given the price schedule, traders including outside investors who purchase information from the seller decide the sizes of their market orders, that are aggregated with orders of liquidity traders, and submitted to the market maker. Price of the security is set by the market maker according to the price schedule. Finally, \tilde{v} is realized and trading is ended as payoff is given to the market participants.

The properties of the random variables and the number of market participants including the outside investors getting informed by the purchase of information are common knowledge, and every agent in the model is assumed to be risk-neutral.

Since there are sufficiently many outside investors who are potential clients for the seller, the equilibrium price of information is uniquely determined such that the expected trading profit of each client equals the price that he pays for the information as the seller's information is auctioned off to outside investors. The outside investors who choose to become clients of information seller pay the price, and then they privately observe the information that they purchase. Subsequently, they base their trading strategies on the purchased information. The clients of the information seller

purchase the unrestricted use of the seller's information in trading securities, but resale of the information is assumed to be prohibited.⁸

A competitive market maker announces the price schedule on the basis of all the public information available including the numbers of buyers of information on each component of \tilde{v} , and the levels of noise added to $\tilde{\theta}$ or $\tilde{\varepsilon}$. Then, traders place their market orders to the market maker, who takes the aggregate net trading order to clear the market and sets the price such that he expects to earn zero profits. The market maker is assumed to observe only the aggregate net trading order, denoted \tilde{y} , and not the individual trading orders submitted. Following the strategic trading model of Kyle (1985) and Admati and Pfleiderer (1988b), the price schedule set by the market maker satisfies the following equation thanks to the zero expected profits condition induced by the competition on the trading floor:

$$P = \bar{v} + \lambda \tilde{y} = E[\tilde{v} | \tilde{y}]. \quad (4)$$

Information seller, analysts and clients of the information seller are information-based traders who trade on their private information to earn trading profits. λ is a measure of market liquidity and it represents how sensitively price moves as net trading order submitted to the market maker changes. The equilibrium λ is determined by the number of different types of information-based traders and the precision of their information. The information seller is a leader of this trading game in that she is able to affect the equilibrium λ , and consequently influence the trading strategy of all the information-based traders, and *ex ante* trading profits of theirs. The instrument she uses to do this is the information sales policy that she announces for access to her information.

⁸While the information sellers are likely to be established financial institutions with reputation and long-term relationship with their current and future customers, the clients of information seller tend to be general investors who cannot credibly convince other investors of the quality of information they try to resell. Since the analysis of this paper is conducted on the basis of one-period model in which the information is assumed to be short-lived, in addition to the clients' lack of credibility, it is conceivable that the information buyers cannot have enough time to resell their purchased information to other investors before trading begins.

The Nash equilibrium of this game follows from the analysis in Kyle (1985) and Admati and Pfleiderer (1988b). The equilibrium consists of (1) information sales policy and a trading strategy for the manager that maximize expected profits when the price functional is taken as given; (2) trading strategies for the analysts that maximize their expected profits when the price functional and trading strategies of other traders are taken as given; (3) decision on information purchase, and trading strategy based on purchased information of outside investors; and (4) a price functional for the market makers such that they obtain a zero expected profit for every realization.

The model presented in this section has a couple of important characteristics, which leads to the results of this paper. First, in this model, the seller's information cannot be leaked to non-clients before trading commences by being reflected in the price of the risky security. When traders place their market orders, they only observe the price schedule, not the actual price. Therefore, they decide the size of their trading orders only on the basis of their own information or their liquidity demand, taking into account the effects of their trading orders on the price.

Second, the price cannot fully reflect all traders' private information, and the market can never collapse due to the presence of the traders with perfect information. This is because the market maker is not able to distinguish random liquidity demand separately from trading orders of other traders who trade on the basis of their information.

In the following analysis, factors that determine which component of information to be sold will be analyzed, and optimal sale strategy is characterized.

III. Information Sales Policy

In this section, we analyze the seller's incentives to sell her private information to other market participants. Following lemma presents market equilibrium given the information sales policy.

Lemma 1

Suppose K outside investors purchase information on $\tilde{\theta}$ with degree of noise (variance) of σ_μ^2 , and information on $\tilde{\varepsilon}$ is sold to

M outside investors with degree of noise of σ_ω^2 , then

1. The equilibrium price schedule is $P - \bar{v} + \lambda \tilde{y}$ where

$$\lambda = \frac{1}{\sigma_u} \sqrt{\frac{(N+1)(\sigma_\theta^2 + 2\sigma_\mu^2)^2 + K\sigma_\theta^2(\sigma_\theta^2 + \sigma_\mu^2)}{((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2 + \frac{(\sigma_\varepsilon^2 + 2\sigma_\omega^2)^2 + M\sigma_\varepsilon^2(\sigma_\varepsilon^2 + \sigma_\omega^2)}{((M+2)\sigma_\varepsilon^2 + 4\sigma_\omega^2)^2} \sigma_\varepsilon^2}.$$

2. Each analyst's expected trading profit, denoted π_a , is

$$\pi_a = \frac{(\sigma_\theta^2 + 2\sigma_\mu^2)^2}{\lambda((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2;$$

3. The expected trading profit of the outside investor purchasing the information on $\tilde{\theta}$, denoted π_θ , is

$$\pi_\theta = \frac{\sigma_\theta^2(\sigma_\theta^2 + 2\sigma_\mu^2)}{\lambda((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2;$$

4. The expected trading profit of the outside investor purchasing the information on $\tilde{\varepsilon}$, denoted π_ε , is

$$\pi_\varepsilon = \frac{\sigma_\varepsilon^2(\sigma_\varepsilon^2 + \sigma_\omega^2)}{\lambda((M+2)\sigma_\varepsilon^2 + 4\sigma_\omega^2)^2} \sigma_\varepsilon^2;$$

5. The expected trading profit earned by the information seller, denoted π_s , is.

$$\pi_s = \frac{(\sigma_\theta^2 + 2\sigma_\mu^2)^2}{\lambda((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2 + \frac{(\sigma_\varepsilon^2 + 2\sigma_\omega^2)^2}{\lambda((M+2)\sigma_\varepsilon^2 + 4\sigma_\omega^2)^2} \sigma_\varepsilon^2.$$

As shown in Lemma 1, information seller earns her trading profit from the information on asset component as well as on growth component. Since analysts also have access to the information on asset component without any noise, seller's trading profit earned from the information on asset component is exactly same as the

analyst's trading profit.

Information seller, analysts and information buyers are all informed traders who expect to earn trading profit from their information on \tilde{v} . Their combined total trading profits are given in the following equation.

$$\begin{aligned}
 \Pi^T &= \pi_a + K \pi_\theta + M \pi_\varepsilon + \pi_s \\
 &= \left[2 \frac{(\sigma_\theta^2 + 2 \sigma_u^2)^2}{\lambda((N+K+2) \sigma_\theta^2 + 2(N+2) \sigma_u^2)^2} \sigma_\theta^2 \right. \\
 &\quad \left. + K \frac{\sigma_\theta^2 (\sigma_\theta^2 + \sigma_u^2)}{\lambda((N+K+2) \sigma_\theta^2 + 2(N+2) \sigma_u^2)^2} \sigma_\theta^2 \right] \\
 &\quad + \left[\frac{\sigma_\varepsilon^2 (\sigma_\varepsilon^2 + \sigma_w^2)}{\lambda((M+2) \sigma_\varepsilon^2 + 4 \sigma_w^2)^2} \sigma_\varepsilon^2 + \frac{(\sigma_\varepsilon^2 + 2 \sigma_w^2)^2}{\lambda((M+2) \sigma_\varepsilon^2 + 4 \sigma_w^2)^2} \sigma_\varepsilon^2 \right]
 \end{aligned} \tag{5}$$

First two items in equation (5) are trading profits earned by information seller, analysts and information buyers from the information on asset component while the last two items are trading profits earned by information seller and information buyers from growth component.

Information seller is a Stackelberg leader of the game in the sense that she effectively determines the equilibrium price schedule and expected trading profits of market participants by pre-committing information sales policy at the beginning of the game. Due to the sufficiently large number of outside investors bidding for the seller's information, in equilibrium prices for the information buyers are equal to the ex ante trading profits earned by the information buyers. That is, information seller fully extracts trading profit to be earned by her clients. Thus, the seller's objective is to maximize her total profit that consists of her own trading profit and the profit earned from the sales of her information. Specifically, the information seller will maximize

$$\Pi^{STP} = \pi_s + K \pi_\theta + M \pi_\varepsilon \tag{6}$$

by optimally choosing $(K, M, \sigma_\theta^2, \sigma_\varepsilon^2)$. Following proposition demon-

strates which component of information will be sold to maximize the seller's total profit.

Proposition 1

A monopolistic information seller will never find it optimal to sell her information on growth component, but her total profit will be maximized by selling her information on asset component to finite number of outside investors.

Proposition 1 shows that the information seller does not sell the information on growth component that she has monopolistic ownership of, while she is able to earn greater total profit by selling the information on asset component that she shares with analysts.

Since the information seller is the monopolistic owner of the information on growth component, we can see from equations (5) that the seller takes all the expected trading profit earned from the information on growth component. The sale of information on growth component creates unnecessary competition between her clients and the seller herself, and the profit from the sale of information on growth component always falls short of the seller's profit which could be earned by trading on her information on growth component without selling it. Therefore, it is desirable for her to keep the information on growth component to herself and trade on it instead of selling it. The intuition here leads to the same outcome as in the Cournot oligopoly model where industry profits are decreasing in a number of identical firms.

However, the seller does not have monopolistic ownership of the information on asset component, and she has to face competition from analysts and share the trading profit from the information on asset component with analysts. Since the seller is not able to appropriate the entire trading profit the information on asset component by herself, her objective now is to maximize not the total trading profit from the information on asset component, but her share of it.

The sale of information to clients who will then optimally use the information to decide their trading strategies has an important commitment effect in that it credibly commits the information seller to a strategy that would not be credible if she were to avoid such sales. The sale of information to a number of clients provides for

more intense competition in the financial market. While this does reduce the total trading profits from asset component, it also has the effect of making the trading strategies of analysts less aggressive. This is actually the 'strategic substitutability' in the sense of Bulow, Geanakoplos, and Klemperer (1985). Although the reduction in overall trading profits from asset component may be substantial, the seller of information does not bear the full cost of its reduction. By selling her information to a number of clients, the information seller is able to make the trading strategy of the analysts less aggressive and increase her share of the trading profits from asset component at the expense of the analysts' profit. It is this strategic externality that makes the sales of information an optimal strategy for the information seller.

In Admati and Pfleiderer (1988a), only a risk averse information seller chooses to sell her information for the purpose of better risk sharing with her clients. As shown in Proposition 1, however, even with a risk neutral information seller, the presence of other information owners in the market justifies her decision to sell her information, and she obtains higher profits by committing herself to a strategy that promotes more intense competition in the market.⁹ Although it is assumed that the seller observes \tilde{v} without any noise, the seller's decision to sell her information depends not on the precision of her information, but on the strategic effect of selling her information which makes the trading strategy of the analysts less aggressive. In particular, even if the seller's information is coarser than the analysts' information, she will still choose to sell her information since this promotes the information-based competition in the market, by which she is able to obtain higher profits.

Since the seller herself is also a trader in the market, as information with lower degree of noise is provided, there might exist a tradeoff between her own trading profit and the profit from information sales. By creating more intense competition in the

⁹This is quite a contrast to Admati and Pfleiderer (1988a). They demonstrate that if either information seller or outsider investor is sufficiently risk tolerant (i.e., close to risk neutral), it is optimal to have only one informed trader in the market. However, Proposition 1 implies that if the seller and outside investors are risk neutral, it is always optimal to increase the number of informed traders in the market unless the seller is the monopolistic owner of the information.

market, the seller's own trading profit might suffer. In order to maximize her total profit from trading and information sales, the seller may have an incentive to sell information higher degree of variance by adding noise to her information before selling it. As the next proposition shows, in spite of this tradeoff, the seller's profit is maximized by selling her information 'as is' without adding any noise to it.

Proposition 2

The information seller never finds it optimal to add any noise to her information on asset component before selling it.

From Proposition 2, we can see that information seller now chooses optimal K^* to maximize her total profit given in the following equation.

$$\Pi^{STP} = \frac{\sigma_\varepsilon^2}{4\lambda} + \frac{(K+1)\sigma_\theta^2}{\lambda(N+K+2)^2} \quad \text{where} \quad \lambda = \frac{1}{\sigma_u} \sqrt{\frac{\sigma_\varepsilon^2}{4} + \frac{(N+K+1)\sigma_\theta^2}{(N+K+2)^2}}. \quad (7)$$

Proposition 2 is quite a contrast to Admati and Pfleiderer (1986) in which the seller with very precise information prefers to add personalized noise to her information before selling it. In their paper, based on a rational expectations model, the information purchased by clients is leaked to non-clients by being reflected in the price before trading commences. As more precise information is sold by the seller, the price carries more of her information due to her clients' more aggressive use of it, causing faster deterioration of its value and even market breakdown. To prevent market collapse, the precision of the information sold to the clients needs to be lower than a critical level. Since more noise needs to be added as the seller's information gets better, the seller cannot fully exploit her improved information for her profit. Since the added noise terms are independent random variables, they observe different signals and submit different sized trading orders to the market maker although information sold to the seller's clients has the same precision. Therefore, it is even possible that some clients make *ex post* trading profits while others suffer *ex post* losses.

In this model, however there is no leakage of the seller's information to other market participants before trading commences,

and price cannot fully carry traders' private information thanks to the random liquidity demand which is exogenous noise in the market. As more precise information is sold to the seller's clients, the value of information increases without ever causing market collapse. Therefore, the seller is able to sell the best information she possibly can, and obtains the highest profit possible by selling her information 'as is' without adding any noise.

Proposition 2 is a lot stronger than a related result in Admati and Pfleiderer (1988a) and Fishman and Hagerty (1995) where the seller is restricted not to dilute her information in the sense that Proposition 2 demonstrates that even if there is no such restriction imposed on the seller, and she is allowed to dilute her information before selling it while she trades on her own information, the seller still never finds it optimal to dilute her information before selling it.

There are a couple of important implications derived from Proposition 2. First, the seller trades on exactly the same information as that sold to her clients. This implies that her expected trading profit is equal to that of each of her clients' which is the price she charges for the information. Therefore, the total profit she expects to make by selling her information to K clients and trading on her own account is exactly same as that by selling her information to $K+1$ clients without being engaged in any trading herself. In equilibrium, as far as the seller communicates honestly there are the same number of traders in the market who trade on the seller's information whether the seller utilizes both options or not.

Second, if the statistical properties of the seller's information and her decision on the sale of her information are common knowledge, one of the incentive problems on the part of the information seller can be avoided. Since the seller trades after she is paid for her information, in order to increase her trading profit, she has an incentive to cheat her clients by actually providing the information with lower precision than the one for which they pay. Suppose the seller is restricted to choosing between selling her information and trading on it, and not allowed to do both. Proposition 2 shows that as long as the seller communicates honestly with her clients, this restriction does not change the seller's total profit. Since the seller no longer trades when she sells her information, she has no reason to cheat her clients, and her information is communicated honestly. Thus, this model is able to provide a strong answer to the question

of why an information possessor may abjure trading totally and sell information only. This contradicts the central result in Allen (1990). Note, however, that this paper has not dealt with the general moral hazard problem in Allen (1990).

Comparative statics on the equilibrium are collected in the following proposition.

Proposition 3

1. The information seller sells her information on asset component to more outside investors as the number of analysts increases.
2. As σ_θ^2 decreases, or σ_ε^2 increases, information on asset component is sold to more outside investors.

As more analysts trade on their information, they collectively trade more aggressively, which reduces the seller's information sales profit. She is unable to recover all of the loss incurred by the more aggressive trading of analysts, but she can still retrieve part of the loss by selling her information to more clients at a lower price, diluting the analysts' trading profits, and thereby enhancing her share of the market trading profits.

As σ_ε^2 increases, information seller earns greater portion of her total profit from the trading profit from the information on growth component, and the seller is more willing to increase her trading profit from growth component at the cost of the profit from asset component to maximize her total profit. This can be achieved by selling the information on asset component to more clients, *i.e.*, and thereby decreasing equilibrium λ , as can be seen from equation (7). The opposite is true in the case that σ_θ^2 increases. To protect her profit from the asset component, the seller is less willing to dilute the value of her information on asset component, and thereby she tends to sell her information to smaller number of clients.

Analysis has been conducted to this section based on the assumption that the seller is able to fully extract the her clients expected trading profit by taking advantage of their bidding competition for the her information. As this assumption is relaxed and the seller cannot possibly extract the 100 percent of her clients expected trading profit, next section shows that optimal use of seller's information is neither trading on it nor directly selling to outside investors.

IV. To Sell or Donate Information

In this section we relax the assumption that the seller is able to fully extract her clients' expected trading profit. Instead, the seller earns $\beta\pi_\theta$ from each client, where π_θ is the expected trading profit earned by each client and $\beta < 1$ is the result of a transaction costs incurred by the seller.¹⁰ Other aspects of the model remain the same. From equation (7), the seller's total profit in this case is given in the following equation.

$$\Pi^{STP} = \frac{\sigma_\varepsilon^2}{4\lambda} + \frac{\sigma_\theta^2}{\lambda(N+K+2)^2} + \frac{\beta K \sigma_\theta^2}{\lambda(N+K+2)^2}$$

$$\text{where } \lambda = \frac{1}{\sigma_u} \sqrt{\frac{\sigma_\varepsilon^2}{4} + \frac{(N+K+1)\sigma_\theta^2}{(N+K+2)^2}}. \quad (8)$$

Next proposition demonstrates that if the seller can keep only a small portion of her clients' expected trading profit as her profit from the sales of information, and the value of the information that she has monopolistic ownership is greater than the one that she shares with analysts, then her total profit can be maximized by simply giving away her information on asset component to outside investors. If the seller opts to use her information in this way, we call it *donation of information* since the seller reveals her information to outside investors without charging any price.

Proposition 4

The donation of information brings higher expected profits for the information seller than sale of information if $\beta < 1/2$ and $\sigma_\varepsilon^2/\sigma_\theta^2$ is high enough. However, if $\beta > 1/2$ and $N > 2$, then sale of information is always superior to the donation of information.

¹⁰If the seller has to pay tax for the profit she earns from the sales of her information, or constant marginal cost is incurred by the seller for each of her client, then she cannot retain 100 percent of the client's expected trading profit as her profit from the sales of information even though the price paid by the information buyers is still equal to their expected trading profit.

The first two terms of equation (8) are the seller's *trading profit* from the growth and the asset component, respectively. The expected trading profit earned by each client is $\pi_{\theta} - \sigma_{\theta}^2 / \lambda(N+K+2)^2$, and the last term of equation (8) is the seller's profit from selling the information on the asset component. The seller's problem is to determine the optimal K^* . We can also see from equation (8) that $K \rightarrow \infty$ is equivalent to the full disclosure of the asset component. In that case, $\pi_{\theta} = 0$ and the seller does not charge any price for her information, and earn any benefit from selling.

An information seller may choose to disclose or donate information to the market even though there is the option of selling it at a strictly positive price. The intuition for the result is more obvious close to the limit. For example, suppose the price that can be charged by the newsletter is bounded by a small quantity (*i.e.* β is small). In this case, most of the expected profit from sales is retained by the buyers of information. An increase in the number of buyers reduces the profits from the asset component, but due to a previously explained intuition for Proposition (3), it increases the profit in the growth component. For a small β , the increase in profits of the growth component outweighs the decrease in the asset component and, thus, full disclosure ($K^* \rightarrow \infty$) is optimal.¹¹

V. Conclusion

This paper analyzes optimal use of private information on financial securities, and investigates the conditions under which a possessor of valuable information on financial security may prefer to sell it directly or give it away free of charge *i.e.*, donating it to other market participants instead of trading on it. A market participant will never find it optimal to sell or donate the information that s/he has monopolistic ownership of. But, if other market participants also have access to the information, sale or donation of information has an important commitment effect in that it credibly commits a risk neutral possessor of information to a strategy which promotes more intense competition among informed traders in the market and makes the trading strategies of other

¹¹When there are finite number of outside investors in the market, then the seller sells her information on asset component to all these outside investors by charging a sufficiently low price.

informed traders less aggressive. It is this strategic externality that makes the selling or donation of information an optimal strategy. Since price cannot reflect all the private information held by traders in the market, information sellers find it optimal to sell their information 'as is' without adding any noise before selling it to clients.

Extreme case of direct sales of information is simply giving away free of charge *i.e.*, donating private information on financial security. It is shown that if the information seller is able to extract only small portion of the information buyers' trading profit, and the profit from the sales of information constitutes a relatively minor part of the seller's total from her information, then strategic effect previously discussed will be maximized by the donation of information.

This paper explores only the issues raised by the direct sales of information. There are many other ways in which a possessor of valuable information may offer it for use in trading. For instance, mutual fund managers sometimes claim to invest their shareholders' money based on private information and research, but shareholders of a mutual fund never directly observe this information. A broader comparison between such different selling methods is much needed, and this paper is best viewed as a first step. On the other hand, information sharing can be conducted when investors with private information on risky security form an exclusive group among themselves and reveal their private information to each other before trading commences, and then trade the risky security based on shared information. Analysis of these two possible use of information are left for further study.

Another important issue in this context concerns the incentive problems of sellers. It is assumed both that the statistical properties of a seller's information are common knowledge, and that truthful communication can be guaranteed. This paper demonstrates, however, that a risk neutral information seller need not trade to maximize her profit if her information is truthfully communicated. This, clearly, is not a complete solution to the general incentive problems. A more detailed appraisal of these issues in an integrated framework remains a topic for further research.

Appendix

Proof of Lemma 1: Suppose a linear price schedule of $P = \bar{v} + \lambda \tilde{y}$ is announced by the market maker. We are going to prove that this price schedule and the following linear trading strategies of information seller, analysts and information buyers form a Nash equilibrium.

• Information seller:

$$\phi \tilde{\theta} + \gamma \tilde{\varepsilon} = \frac{\sigma_{\theta}^2 + 2 \sigma_{\mu}^2}{\lambda((K+N+2) \sigma_{\theta}^2 + 2(N+2) \sigma_{\mu}^2)} \tilde{\theta} + \frac{\sigma_{\varepsilon}^2 + 2 \sigma_{\omega}^2}{\lambda((M+2) \sigma_{\varepsilon}^2 + 4 \sigma_{\omega}^2)} \tilde{\varepsilon},$$

• Analysts:

$$\psi \tilde{\theta} = \frac{\sigma_{\theta}^2 + 2 \sigma_{\mu}^2}{\lambda((K+N+2) \sigma_{\theta}^2 + 2(N+2) \sigma_{\mu}^2)} \tilde{\theta},$$

• Buyers of information on asset component:

$$\alpha(\tilde{\theta} + \tilde{\mu}_i) = \frac{\sigma_{\theta}^2}{\lambda((K+N+2) \sigma_{\theta}^2 + 2(N+2) \sigma_{\mu}^2)} (\tilde{\theta} + \tilde{\mu}_i),$$

• Buyers of information on growth component:

$$\tau(\tilde{\varepsilon} + \tilde{\omega}_j) = \frac{\sigma_{\varepsilon}^2}{\lambda((M+2) \sigma_{\varepsilon}^2 + 4 \sigma_{\omega}^2)} (\tilde{\varepsilon} + \tilde{\omega}_j).$$

Taking the price schedule and other informed traders' trading strategies as given, the seller's optimal trading strategy is derived from following maximization problem given the information of $\tilde{\theta}$ and $\tilde{\varepsilon}$:

$$\max_x E[x(\bar{v} + \tilde{\theta} + \tilde{\varepsilon} - \bar{v} - \lambda(x + N \phi \tilde{\theta} + \sum_{i=1}^K \alpha(\tilde{\theta} + \tilde{\mu}_i) + \sum_{j=1}^M \tau(\tilde{\varepsilon} + \tilde{\omega}_j) + \tilde{u})) | \tilde{\theta}, \tilde{\varepsilon}].$$

The first order condition is given by:

$$\tilde{\theta} + \tilde{\varepsilon} - 2 \lambda x - \lambda N \phi \tilde{\theta} - \lambda K \alpha \tilde{\theta} - \lambda M \tau \tilde{\varepsilon} = 0. \quad (\text{A.1})$$

Taking the price schedule and other informed traders' trading strategies as given, the analyst's optimal trading strategy is derived from following maximization problem given the information of $\tilde{\varepsilon}$:

$$\max_z E[z(\bar{v} + \tilde{\theta} + \tilde{\varepsilon} - \bar{v} - \lambda(\phi\tilde{\theta} + \gamma\tilde{\varepsilon} + z + (N-1)\phi\tilde{\theta} + \sum_{i=1}^K \alpha(\tilde{\theta} + \tilde{\mu}_i) + \sum_{j=1}^M \tau(\tilde{\varepsilon} + \tilde{\omega}_j) + \tilde{u})) | \tilde{\theta}],$$

The first order condition is given by:

$$\tilde{\theta} - \lambda\phi\tilde{\theta} - 2\lambda z - \lambda(N-1)\phi\tilde{\theta} - \lambda K\alpha\tilde{\theta} = 0. \quad (\text{A.2})$$

Given information on $\tilde{\theta}$, each outside investor buying information on asset component solves the following maximization problem taking price schedule, and other traders' trading strategies as given:

$$\max_w E[w(\bar{v} + \tilde{\theta} + \tilde{\varepsilon} - \bar{v} - \lambda(\phi\tilde{\theta} + \gamma\tilde{\varepsilon} + N\phi\tilde{\theta} + w + \sum_{i \neq h}^K \alpha(\tilde{\theta} + \tilde{\mu}_i) + \sum_{j=1}^M \tau(\tilde{\varepsilon} + \tilde{\omega}_j) + \tilde{u})) | \tilde{\theta} + \tilde{\mu}_h],$$

The first order condition is given by:

$$\begin{aligned} & \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\mu}^2} (\tilde{\theta} + \tilde{\mu}_h) - \lambda\phi \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\mu}^2} (\tilde{\theta} + \tilde{\mu}_h) - \lambda N\phi \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\mu}^2} (\tilde{\theta} + \tilde{\mu}_h) - 2\lambda w \\ & - \lambda(K-1)\alpha \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\mu}^2} (\tilde{\theta} + \tilde{\mu}_h) = 0. \end{aligned} \quad (\text{A.3})$$

Given information on $\tilde{\varepsilon}$, each outside investor buying information on growth component solves the following maximization problem taking price schedule, and other traders' trading strategies as given:

$$\begin{aligned} & \max_t E[t(\bar{v} + \tilde{\theta} + \tilde{\varepsilon} - \bar{v} - \lambda(\phi\tilde{\theta} + \gamma\tilde{\varepsilon} + N\phi\tilde{\theta} + \sum_{i=1}^K \alpha(\tilde{\theta} + \tilde{\mu}_i) + t \\ & + \sum_{j=1}^M \tau(\tilde{\varepsilon} + \tilde{\omega}_j) + \tilde{u})) | \tilde{\varepsilon} + \tilde{\omega}_h]. \end{aligned}$$

The first order condition is given by:

$$\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\nu'}^2} (\tilde{\varepsilon} + \tilde{\omega}_i) - \lambda \gamma \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\omega'}^2} (\tilde{\varepsilon} + \tilde{\omega}_i) - 2\lambda t - \lambda(M-1) \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\omega'}^2} (\tilde{\varepsilon} + \tilde{\omega}_i) = 0. \quad (\text{A.4})$$

From equations (A.1), (A.2), (A.3), and (A.4) we can have following set of simultaneous equations, and by solving these equations we can derive optimal trading strategies of informed traders.

$$2\lambda\phi + N\lambda\psi + K\lambda\alpha = 1$$

$$2\lambda\gamma + M\lambda\tau = 1$$

$$\lambda\phi + (N+1)\lambda\psi + K\lambda\alpha = 1$$

$$\lambda\phi \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\mu^2} + N\lambda\psi \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\mu^2} + 2\lambda\alpha + (K-1)\lambda \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\mu^2} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\mu^2}$$

$$\lambda\gamma \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\nu'}^2} + 2\lambda\tau + (M-1)\lambda \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\nu'}^2} = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\nu'}^2}.$$

Given price schedule, the net aggregate trading order submitted to the market maker is given in the following equation:

$$\tilde{y} - \phi\tilde{\theta} + \gamma\tilde{\varepsilon} + N\psi\tilde{\theta} + \sum_{i=1}^K \alpha(\tilde{\theta} + \tilde{\mu}_i) + \sum_{j=1}^M \tau(\tilde{\varepsilon} + \tilde{\omega}_j) + \tilde{u}. \quad (\text{A.5})$$

The market maker sets the price schedule that obtains a zero expected profit for each realization of \tilde{y} , and the price schedule satisfies the following conditional expectation:

$$\bar{v} + \lambda\tilde{y} = \bar{v} + E[\tilde{\theta} + \tilde{\varepsilon} | \tilde{y}]$$

Expected trading profit earned by the information seller given in the Lemma 1 is derived in the following equation given equilibrium trading strategies of informed traders and price schedule.

$$E[(\phi\tilde{\theta} + \gamma\tilde{\varepsilon})(\bar{v} + \tilde{\theta} + \tilde{\varepsilon} - \bar{v} - \lambda\tilde{y})]$$

Expected trading profits of analysts, and information buyers are derived in the similar ways.

Q.E.D.

Proof of Proposition 1: Information seller's total profit is given in the following equation from Lemma 1.

$$\begin{aligned}
 \Pi^{STP} &= \pi_s + K \pi_\theta + M \pi_\varepsilon \\
 &= \frac{(\sigma_\theta^2 + 2 \sigma_\mu^2)^2}{\lambda((N+K+2) \sigma_\theta^2 + 2(N+2) \sigma_\mu^2)^2} \sigma_\theta^2 + \frac{(\sigma_\varepsilon^2 + 2 \sigma_\omega^2)^2}{\lambda((M+2) \sigma_\varepsilon^2 + 4 \sigma_\omega^2)^2} \sigma_\varepsilon^2 \\
 &\quad + K \frac{\sigma_\theta^2 (\sigma_\theta^2 + \sigma_\mu^2)}{\lambda((N+K+2) \sigma_\theta^2 + 2(N+2) \sigma_\mu^2)^2} \sigma_\theta^2 + M \frac{\sigma_\varepsilon^2 (\sigma_\varepsilon^2 + \sigma_\omega^2)}{\lambda((M+2) \sigma_\varepsilon^2 + 4 \sigma_\omega^2)^2} \sigma_\varepsilon^2 \\
 &= \sigma_\mu \sqrt{\frac{\frac{(\sigma_\theta^2 + 2 \sigma_\mu^2)^2 + K \sigma_\theta^2 (\sigma_\theta^2 + \sigma_\mu^2)}{((N+K+2) \sigma_\theta^2 + 2(N+2) \sigma_\mu^2)^2} \sigma_\theta^2 + \frac{(\sigma_\varepsilon^2 + 2 \sigma_\omega^2)^2 + N \sigma_\varepsilon^2 (\sigma_\varepsilon^2 + \sigma_\omega^2)}{((M+2) \sigma_\varepsilon^2 + 4 \sigma_\omega^2)^2} \sigma_\varepsilon^2}{(N+1)(\sigma_\theta^2 + 2 \sigma_\mu^2)^2 + K \sigma_\theta^2 (\sigma_\theta^2 + \sigma_\mu^2)} \sigma_\theta^2 + \frac{(\sigma_\varepsilon^2 + 2 \sigma_\omega^2)^2 + M \sigma_\varepsilon^2 (\sigma_\varepsilon^2 + \sigma_\omega^2)}{((M+2) \sigma_\varepsilon^2 + 4 \sigma_\omega^2)^2} \sigma_\varepsilon^2} \\
 &\hspace{15em} \text{(A.6)}
 \end{aligned}$$

By taking derivative of Π^{STP} with respect to M , we can find Π^{STP} is monotonically decreasing in M , and therefore optimal M is zero. However, if we can find $(\partial \Pi^{STP} / \partial K)|_{K=1} > 0$, and therefore, the seller can earn higher total profit by selling the information on asset component.

Q.E.D.

Proof of Proposition 2: With $M=0$, the seller total profit is given in the following equation from equation (A.6).

$$\Pi^{STP} = \pi_s + K \pi_\theta + M \pi_\varepsilon$$

$$\begin{aligned}
& \frac{(\sigma_\theta^2 + 2\sigma_\mu^2)^2 + K\sigma_\theta^2(\sigma_\theta^2 + \sigma_\mu^2)}{((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2 + \frac{\sigma_\varepsilon^2}{4} \\
& = \sigma_\mu \frac{\sqrt{\frac{(N+1)(\sigma_\theta^2 + 2\sigma_\mu^2)^2 + K\sigma_\theta^2(\sigma_\theta^2 + \sigma_\mu^2)}{((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2 + \frac{\sigma_\varepsilon^2}{4}}}{\sqrt{\frac{(N+1)(\sigma_\theta^2 + 2\sigma_\mu^2)^2 + K\sigma_\theta^2(\sigma_\theta^2 + \sigma_\mu^2)}{((N+K+2)\sigma_\theta^2 + 2(N+2)\sigma_\mu^2)^2} \sigma_\theta^2 + \frac{\sigma_\varepsilon^2}{4}}} \quad (A.7)
\end{aligned}$$

By taking derivative with respect to σ_μ^2 , we can find Π^{STP} is monotonically decreasing, and therefore optimal $\sigma_\mu^2=0$ is obtained.
Q.E.D.

Proof of Proposition 3: With $M=0$, and $\sigma_\mu^2=0$, the seller's total profit is now

$$\Pi^{STP} = \frac{\sigma_\varepsilon^2}{4\lambda} + \frac{(K+1)\sigma_\theta^2}{\lambda(N+K+2)^2} \quad \text{where} \quad \lambda = \frac{1}{\sigma_u} \sqrt{\frac{\sigma_\varepsilon^2}{4} + \frac{(N+K+1)\sigma_\theta^2}{(N+K+2)^2}}.$$

By taking the derivative of Π^{STP} with respect to K , we have

$$\frac{\partial \Pi^{STP}}{\partial K} = \frac{\sigma_\theta^2}{\sigma_u(N+K+2)^3} \left(\frac{\sigma_\varepsilon^2}{4} + \frac{(N+K+1)\sigma_\theta^2}{(N+K+2)^2} \right)^{3/2} [A+B] \quad \text{where} \quad (A.8)$$

$$A = \frac{\sigma_\varepsilon^2}{8} (3N-K)$$

$$B = \frac{\sigma_\theta^2}{2(N+K+2)^2} (-K^2 + K(N-1) + 2N^2 + 3N) \quad (A.9)$$

Optimal K^* is obtained from the solution of $A+B=0$ given in equation (A.8). Since $\partial(A+B)/\partial K < 0$, for all K , second order condition is satisfied. From equation (A.8), we can find $\partial(A+B)/\partial N > 0$, $\partial(A+B)/\partial \sigma_\varepsilon^2 > 0$, and $\partial(A+B)/\partial \sigma_\theta^2 < 0$, and therefore, comparative statics in the proposition are derived.

Q.E.D.

Proof of Proposition 4: By taking the derivative of Π^{STP} in equation (8) with respect to K , we have

$$\frac{\partial \Pi^{STP}}{\partial K} - \frac{\sigma_{\theta}^2}{\sigma_u(N+K+2)^3} \left(\frac{\sigma_{\varepsilon}^2}{4} + \frac{(N+K+1) \sigma_{\theta}^2}{(N+K+2)^2} \right)^{-3/2} [A+B] \quad (\text{A.10})$$

where

$$A = \frac{\sigma_{\varepsilon}^2}{8} [K(1-2\beta) + N(2\beta+1) + 4(\beta-1)]$$

$$B = \frac{\sigma_{\theta}^2 (-\beta K^2 + K(\beta N + 2\beta - 3) + (N+1)(2\beta N + 4(\beta-1) + N))}{2(N+K+2)^2} \quad (\text{A.11})$$

and

$$\frac{\partial A}{\partial K} = \frac{\sigma_{\varepsilon}^2}{8} (1-2\beta)$$

$$\frac{\partial B}{\partial K} = \frac{\sigma_{\theta}^2}{2(N+K+2)^3} (1-\beta(N+2))(3K+3N+2). \quad (\text{A.12})$$

For $\beta \geq 1/2$ and $N \geq 2$, $(\partial A / \partial K) + (\partial B / \partial K) < 0$ holds for all K and from equation (A.11). We can see that $A+B > 0$ at $K=0$ and $K=1$, but as K increases, $A+B$ decreases to $-\infty$. Therefore, there exists a unique $K^* > 1$ such that $(\partial \Pi_s / \partial K)|_{K=K^*} < 0$ and $(\partial^2 \Pi_s / \partial K^2)|_{K=K^*} < 0$, and Π_s is maximized at K^* .

If Π^{STP} increases in $K \geq 0$, i.e., $(\partial \Pi_s / \partial K) \geq 0$, then optimal K^* is $+\infty$ and donation of information is the optimal strategy that maximizes Π_s . For $\beta < 1/2$, we can consider two possibilities.

- $\beta < 1/2$ and $1 - \beta(N+2) > 0$

In this case, from equation (A.12) we have $(\partial A / \partial K) > 0$ and $(\partial B / \partial K) > 0$. Thus, if $A+B > 0$ holds at $K=0$, i.e.,

$$-\frac{\sigma_{\varepsilon}^2}{8} [N(2\beta+1) + 4(\beta-1)] + \frac{\sigma_{\theta}^2}{2(N+2)^2} (N+1)(2\beta N + 4(\beta-1) + N)$$

$$-(N(2\beta+1)+4(\beta-1))\left(\frac{\sigma_\varepsilon^2}{8}+\frac{\sigma_\theta^2}{2(N+2)^2}(N+1)\right)>0$$

then for $N \geq 4$, $(\partial \Pi^{STP}/\partial K) \geq 0$ for all K and $K^* = +\infty$ is obtained.

- $\beta < 1/2$ and $1 - \beta(N+2) < 0$

In this case, $(\partial A/\partial K) > 0$ is still true, and the minimum of A obtained at $K=0$ is

$$\frac{\sigma_\varepsilon^2}{8}(N(2\beta+1)+4(\beta-1)).$$

But $(\partial B/\partial K) < 0$ holds for all K , and the minimum of B is now obtained at $K = +\infty$, which is $-\sigma_\theta^2 \beta$. Therefore, $A+B > 0$ is satisfied for all K if following condition is satisfied.

$$\min A + \min B = \frac{\sigma_\varepsilon^2}{8}(N(2\beta+1)+4(\beta-1)) - \sigma_\theta^2 \beta > 0$$

By combining two cases, we now have sufficient condition for the donation of information.

$$\beta < 1/2 \quad \text{and}$$

$$\frac{\sigma_\varepsilon^2}{8}(N(2\beta+1)+4(\beta-1)) - \sigma_\theta^2 \beta > 0.$$

We can see that for $1 - \beta(N+2) < 0$, $(\sigma_\varepsilon^2/8)(N(2\beta+1)+4(\beta-1)) - \sigma_\theta^2 \beta > 0$ is more likely to hold for bigger N and σ_ε^2 but smaller σ_θ^2 , and the result follows.

Q.E.D.

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